THE ECONOMIC THEORY OF

TECHNOLOGICAL CHANGE

Prof. MARCOS C. CAVALCANTI
DE ALBUQUERQUE

EAESP/FGV

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This article is an attempt to analyse the theoretical implications of technological change within a general framework and then, in the specialized factor-augmenting case. Two-factor production functions are used in the analysis, but the main results are generalized to the n-factor case.

The analysis is done within a framework that allows for biased technological change, and the three main definitions, those according to HICKS, HARROD and SOLOW, are explicitly derived and related to each other.

The results obtained are useful in understanding the phenomenon of technological change and can be of great help in specifying econometric models, such as those based on the new "translog" function.(1)

Let us assume that it is possible to define a macro-economic relation between output and a set of inputs, the aggregate production function.(2) Such a production function, defined in

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(1) See ALBUQUERQUE (1985). This article is a slightly modified version of chapter 2 of "A Translog Analysis of Technological Change and Scale Effects in Brazilian Agriculture: A Case of Inefficient Modernization", a Ph.D. Thesis presented to the Department of Economics of Harvard University.

(2) Defining the function as $F(\underline{X})$ where $\underline{X}$ is the input vector, F is a neoclassical production function if it possesses the following characteristics:

a) F is a continuous function from the set of all non-negative input bundles onto the set of non-negative output levels.
b) F has continuous second order partial derivatives with respect to all arguments.
c) $F_i \; (\underline{X}) \geqslant 0$.
d) $F \; (\lambda \underline{X}) = \lambda F \; (\underline{X})$ for all $\lambda > 0$ and all $\underline{X} \geqslant 0$.
e) Strict quasi-concavity: for any $\underline{X} \geqslant 0$, $\underline{X}' \geqslant 0$, $0 < \lambda < 1$ and for any $C > 0$, if $F \; (\underline{X}) \geqslant C$ and $F \; (\underline{X}') \geqslant C$ then $F \; (\lambda \underline{X} + (1 - \lambda) \; \underline{X}') \geqq C$ with equality if $\underline{X} = \underline{X}'$.

See BURMEISTER et al (1970).

close analogy with the microeconomic production function, can be characterized by 4 parameters which describe its "abstract technology", i.e., the efficiency parameter, the scale parameter, the intensity parameter and the substitution parameter.(3)

A change in "abstract technology" can be associated with a number of factors such as greater possibility of substitution between factors of production, economies or diseconomies of scale, increased education and training, inter-sectorial resource shifts, organizational improvements, etc.. Although some of these reasons may be directly associated with changes in the stock of knowledge, usually resulting from research and development, others may not. Such is the case, for instance, of the process of training which does not really increase the stock of knowledge, but may speed up its diffusion thereby leading to a shift of the production function.(4)

The aggregate production function should not be interpreted as a relationship describing the most efficient production techniques, but rather, as representing the average level of production possibilities available to producers at any given moment in time. Interpreted in this fashion, the aggregate production function is not a totally exogenous element in economic theory, exclusively dependent on technologically determined stock of knowledge, but rather an endogenous variable jointly determined by economic and non-economic considerations. The production function, interpreted as a totally technologically determined concept, restricts its more wide-ranging characteristics, especially in a dynamic context.(5)

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(3) BROWN (1966) chapters 2, 4. As an example, take the 2 factor CES production function where Y is output, K is capital and L is labour, $Y = \gamma \left[\alpha K^{-\rho} + (1-\alpha) L^{-\rho}\right]^{-\lambda/\rho}$; γ is the efficiency parameter, α is the intensity parameter, λ is the scale or degree of homogeneity parameter and ρ is the substitution parameter.

(4) For a discussion of the way in which these factors influence technological change see KENNEDY et al (1972), HEERTJE (1973).

(5) This broader view of the concept of the production function lends itself, for instance, to the theory of induced innovation according to which the adoption of technological progress depends on factor price ratios, even though physically, new techniques of productions may be known well in advance.

The Theory of Technological Change

Technical change, at the aggregate level, can be represented by an index which specifies the production function shifts, generating a whole family of such functions. The index itself is a function of all factors which cause technological change such as economic, technological, cultural, climatic and other factors. As a simplification, we shall associate this index, which we shall call "t", with calendar time. This interpretation of the parameter "t" assumes that the shifts of the aggregate production function occur in a smooth and continuous way, although we should be aware that at the microeconomic level technological change occurs erratically, at times going forward and then retrogressing.

Early discussions of technological progress attempted to evaluate the effect of mechanization on the level of employment and, consequently, on labour's participation in the income generated by an economic system. It was natural that discussions focused on classifying types of technological change according to their effect on income shares accrueing to specific factors.(6)

Three types of classification schemes are the most common and widely used, those of HICKS, HARROD and SOLOW.(7) Each definition says that, for a two input (K and L) production function, technological change is neutral if it leaves factor shares unaltered along paths where the capital-labour ratio, the rate of return on capital, and the wage rate, respectively, are constant. Conversely, technical change is defined as capital-using (the same as labour-saving) or labour-using (the same as capital-saving) according to each definition if, respectively, the share of capital or that of labour rises.

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(6) Usually, classifications of technological progress try to measure its impact on some predetermined variable, i.e., the capital-output ratio, the marginal rate of substitution, factor shares, average labour productivity etc. However, these variables are not dependent on technology alone; they depend also on proportional factor supplies. Thus, it is necessary to isolate the technological effect by specifying a particular path along which is measured the "pure" effect of technological change.

(7) HICKS (1932), HARROD (1948) and SOLOW (1962).

HICKS neutrality, which requires the constancy of relative shares along a path where the capital-labour ratio is constant, is geared to analysing a short-run situation when capital and labour availabilities are fixed.

HARROD's definition allows for long-run adjustments of factor availability, but imposes the restriction of a constant rate of return on capital. This hypothesis is in the spirit of "neo-Keynesian" economics, according to which, capitalists, in mature economies, determine their average rate of profit, which, therefore, ceases to be an exogenously determined variable.

Finally, SOLOW's definition is consistent with an underdeveloped economy where the wage rate, supposedly at subsistence level, cannot be lowered, and is not allowed to be raised by the well-known mechanisms described by the unlimited-supply-of-labour models.[8]

Originally, the three classifications of technological change were not explicitly defined in terms of factor shares. HICKS neutrality required a constant marginal rate of substitution at a given full employment capital-labour ratio; HARROD required a constant capital-output ratio at a given rate of interest; SOLOW required a constant labour-output ratio given a fixed wage rate.

As BECKMAN and SATO (1968) have shown in their attempt to define new types of technical progress, when factor shares are introduced as variables to be considered invariant in relation to capital-labour, capital-output, and labour-output ratios, these definitions turned out to coincide with those given by HICKS, HARROD and SOLOW, respectively. Actually, this can be seen in a fairly straightforward way.

Defining F(.) as the neoclassical production function, Π_K as the share of capital, and dots as total differentiation with respect to time,

$$\Pi_K = \frac{F_K K}{Y} = \frac{rK}{Y} \tag{1}$$

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(8) FEI and RANIS (1965) used SOLOW's definition in their analysis of technological change in underdeveloped economies.

must imply that if technological progress is neutral in HARROD's definition, then $\dot{\Pi}_K = 0 = \dot{\Pi}_L$ since $\dot{\Pi}_K, r$ and K/Y are not all independent. (The SOLOW neutrality case can be easily shown using the expression $\Pi_L = \frac{F_L L}{Y} = \frac{wL}{Y}$). For HICKS neutral technological progress, the equivalence of his original definition to the statement in terms of factor shares can be shown as follows: defining $p = \frac{r}{w}$, and, since cost minimization equates p to the marginal rate of substitution,

$$\frac{\Pi_K}{\Pi_L} = \frac{rK}{WL} = pk \tag{2}$$

which must imply that $\dot{\Pi}_K = 0 = \dot{\Pi}_L$ since Π_K, Π_L, p and k are not all independent. Stating HICKS neutrality in terms of factor shares is convenient in that it establishes the bias of technological change in the n factor case by a single measure, while the original definition, stated in terms of marginal rates of substitution, would lead to n - 1 measures of bias for each factor.[9]

a) The General Case:

Using results obtained by FERGUSON (1971), and by DIAMOND, MCFADDEN, and RODRIGUEZ in FUSS et al (1978), it is possible to derive relationships which clarify the role of technological progress in growth accounting. Beginning with the usual production function

$$Y = F(K,L,t) \tag{3}$$

define the rate of technological progress as[10]

$$T = \frac{F_t}{F} = \frac{KF_{Kt} + LF_{Lt}}{F} \tag{4}$$

and

$$T = \frac{F_t}{F} = \frac{KF_{Kt} + LF_{Lt}}{KF_K + LF_L} \tag{5}$$

(9) The same reasoning holds also for HARROD's and SOLOW's definition of technological change.

(10) A subscript denotes partial differentiation, dots denote total time derivative and hats denote logarithmic differentiations with respect to time.

Following HICKS' original definition, let B represent the bias of technological progress.

$$B = \frac{\partial \ln M}{\partial t} = \frac{\partial\left(\frac{F_K}{F_L}\right)}{\partial t}\,\frac{F_L}{F_K} = \frac{F_{Kt}}{F_K} - \frac{F_{Lt}}{F_L} \tag{6}$$

where M denotes the marginal rate of technical substitution between capital and labour.(11) Technological progress is HICKS capital using (labour saving), neutral, or labour using (capital saving) depending on whether $B \gtreqless 0$.

Using expression (5) and (6), it is possible to express the total rate of change of the factor marginal products in terms of the rate of technological changes, the bias of technological progress and of factor shares.(12)

$$\hat{F} = T + \Pi_K \hat{K} + \Pi_L \hat{L} = T + \Pi_K \hat{k} + \hat{L} \;. \tag{7}$$

$$\hat{r} = T + \Pi_L \left(B - \frac{\hat{k}}{\sigma}\right) \tag{8}$$

$$\hat{w} = T - \Pi_K \left(B - \frac{\hat{k}}{\sigma}\right) \tag{9}$$

$$\hat{S} = \widehat{\left(\frac{\Pi_K}{\Pi_L}\right)} = B + \left(1 - \frac{1}{\sigma}\right) \hat{k} \;, \tag{10}$$

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(11) Alternatively, using the production function in the intensive form

$$B = \frac{\partial \ln M}{\partial t} = \frac{\partial \ln}{\partial t}\left(\frac{f_k}{f - kf_k}\right) = \frac{f_{kt}f - f_k f_t}{f_k\,(f - kf_k)}$$

(12) For a detailed treatment of this matter see appendix. In arriving at expressions we assumed linear homogeneity and profit maximizing behaviour in a competitive market. We assume linear homogeneity for heuristic purposes, since making this assumption allows us to derive σ, the elasticity of factor substitution. Were F (.) not homogeneous σ would have to be replaced by some expression, also a function of the same variables that determine σ, which would still reflect "factor substitution", although not in the conventional form expressed by the elasticity of substitution.

where Π_i is the share of factor i in total income, σ is the elasticity of substitution, hats denote logarithmic differentia tion, r is the return of capital, w is the wage rate and S is the relative factor share.

Some important conclusions can be drawn from the relationships derived above which shed light on the importance of technological change in determining the values of some key variables in an economic system.

The growth rate of output can be decomposed into two separate parts: a) growth of output attributable to technological progress alone and which results exclusively from shifts in the production function, and b) growth of output attributable to increased supply of factors of production and which results exclusively from movements along the static production function. In figure 1 the former growth effect is represented by a movement from B. to C, while the latter is represented by a move-ment from A to B.

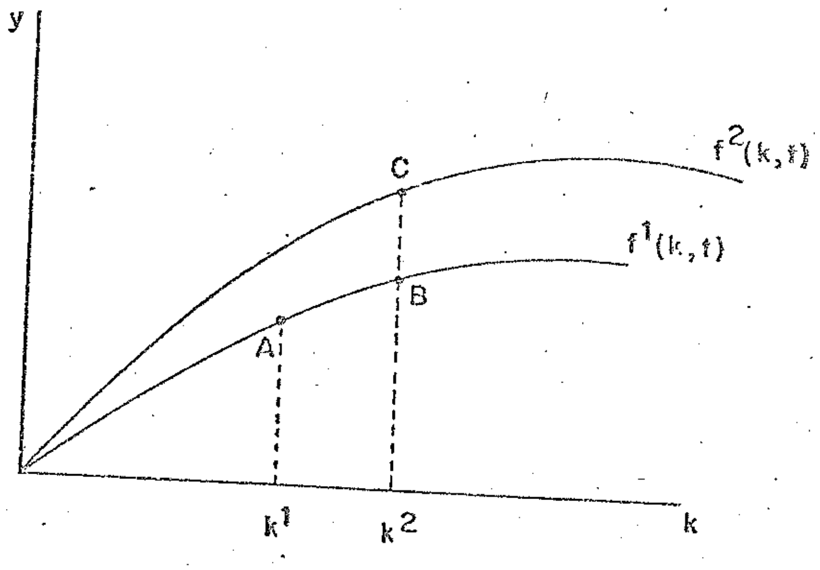


Figure 1 THE TECHNOLOGICAL AND THE FACTORIAL COMPONENTS OF GROWTH OF OUTPUT

As a result of the existence of two growth components, factor price growth equations and the relative factor share growth equation are also dependent on technological and factorial components. Actually, growth of factor prices (8) and (9) are dependent on four different variables, namely the rate of technological progress (T), the bias of technological progress (B), the growth rate of factors of production ($\hat{k}$) and the elasticity of substitution (σ). The variables T, B and σ are technological components, whereas $\hat{k}$ is the factorial component.

From equations (8) and (9) it can be seen that the rate of technological progress T has a positive effect on both factor prices since it increases their marginal products. On the other hand, the other growth components affect factor prices in different directions.

The HICKS bias, B, will increase the growth of the rate of return on capital if technological progress is capital-using (B>0). Conversely, it will tend to reduce it if technological progress is labour-using (B<0). Mutatis mutandis, B will affect the growth of the wage rate. Of course, B has no effect on factor prices if technological progress is HICKS neutral (B = 0).

The expression $\frac{\hat{k}}{\sigma}$ in equations (8) and (9) shows the effect of changes in factor intensity "corrected" for the possibility of factor substitution in production. The elasticity of substitution can either dampen or enhance the effect of factor changes on growth of factor prices depending on whether $\sigma \gtreqless 1$. If $\sigma > 1$, the effect (positive or negative) of changes in factor intensity on factor prices will be dampened since the ease of factor substitution implied by greater-than-unity elasticity of substitution will partly compensate the influence of changes in factor supplies on factor prices. (13) If $\sigma < 1$, the effects of weak substitution possibilities will increase the effect of changed factor supplies on factor prices. Of course, if $\sigma = 1$ there is perfect substitution, and changes in factor supplies affect factor prices without any interference from factor substitution possibilities. It is clear, therefore, that no unambiguous conclusion about effects of technological change on factor rewards can be drawn without taking factor supplies and factor substitution into consideration.

It is possible, for example, that a capital using technological change $(B>0)$ may be associated with a fall in the rate of return on capital if $\Pi_L \frac{\hat{k}}{\sigma} > T + \Pi_L B$; similarly, a labour using technological change $(B < 0)$ can be associated with an increase in the rate of return on capital if $T > | \Pi_L (B - \hat{K}/\sigma) |$. (14)

All we can say is that assuming $\hat{k} = 0$ (i.e. factor supplies either do not increase, or increase in equal proportion) a capital-using technological change will always increase the rate of return on capital and decrease the wage rate, and that a labour-using technological change will have the opposite effect.

Here again, it is important to point out to the difference between the macro and the micro approaches. While in the short-run, at the macro level, it is conceivable that $\hat{k} = 0$, the factor intensity k is never a constant at the level of the individual firm under competitive assumptions; the

(13) The substitution effect on productivity theory is always negative.

(14) We assume T is positive; in other words, there is no technological retrogression.

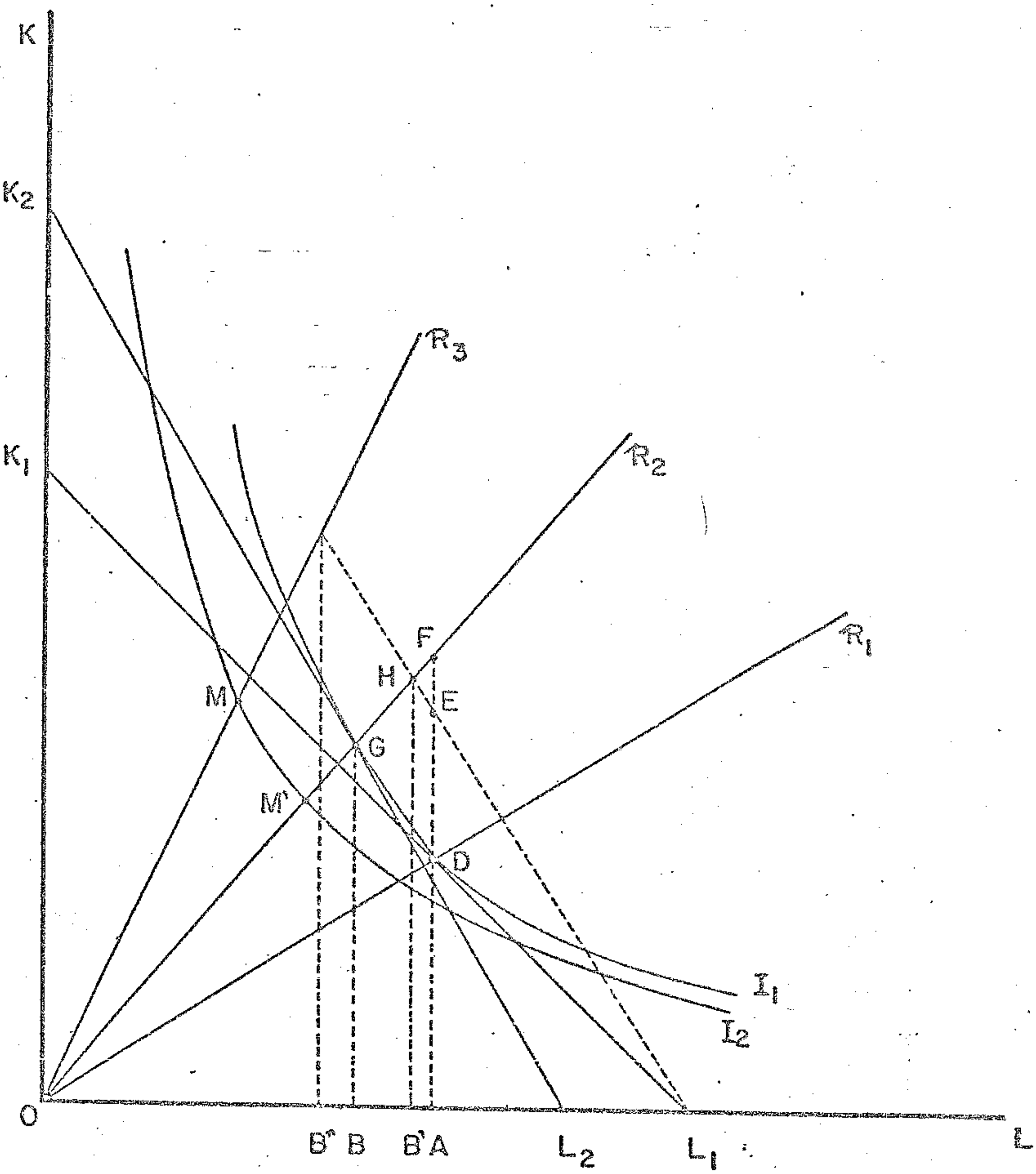
individual firm, attempting to minimize its costs, will always change its capital-labour ratio as a result of changes in factor prices caused by biased technological progress.

Finally, equation (10) states that the share of capital will increase relative to that of labour, assuming $\hat{k} = 0$, if technological bias is capital-using, and vice-versa, if it is labour-using.

If factor intensities change, its "corrected" effect $(1 - \frac{1}{\sigma})\ \hat{k}$, precludes us from drawing any a priori conclusions about the effects of technological change on factor shares without taking specific values for all variables into consideration. Depending on the value of σ , the elasticity of substitution, even a capital-using technological change may be associated with a fall in the relative share of capital. This would be the case if, for example, capital intensity increases $(\hat{k} > 0)$ and factor substitution is difficult $(\sigma < 1)$; as a result, the rate of return on capital falls (the lower the elasticity of substitution, the greater the intensity of the fall), and if the capital-using bias of technological progress is not strong enough to offset it, $(B < | 1 - \frac{1}{\sigma})\ \hat{k}\ |)$ the relative share of capital will fall relative to that of labour.

These relations can be shown graphically as follows. Initially, equilibrium is attained at point D, in figure 2 with the profit maximizing capital-labour ratio given by k_1, and relative prices given by the slope of line K_1L_1. Relative shares $(\frac{\Pi_K}{\Pi_L})$ measured in terms of labour units equals $AL_1/0A$. Suppose now, that given a macroeconomic increase in the aggregate capital-labour ratio, relative factor prices change to equal the slope of line K_2L_2. The new equilibrium would shift to point G, were relative shares would equal $BL_2/0B$ (we

Figure 2 BIASED AND UNBIASED TECHNICAL PROGRESS



are assuming no technological progress, i.e. isoquant I_1 does not shift).

The percentage change in the capital-labour ratio can be identified as $\hat{k} = (\frac{FA}{0A} - \frac{DA}{0A}) \div \frac{DA}{0A} = \frac{FD}{DA}$. Furthermore, drawing the auxiliary line HL_1, parallel to K_2L_2, allows the identification of the percentage change in the factor price ratio as $\hat{p} = (\frac{EA}{AL_1} - \frac{DA}{AL_1}) \div \frac{DA}{AL_1} = \frac{ED}{DA}$. Since the elasticity of substitution equals $\hat{k}/\hat{p}$, $\sigma = \frac{FD}{ED}$.

In this particular case we can see that since $\sigma > 1$ and $\hat{k} > 0$, the percentage change in relative shares was positive. Using the properties of the similar triangles OGL_2 and OHL_1, it can be seen that, indeed, relative shares increased from $\frac{AL_1}{0A}$ to $\frac{B'L_1}{0B'} = \frac{BL_2}{0B}$.

Of course, the opposite would have occurred, had the elasticity of substitution been smaller than unity.

It can also be observed that, had the elasticity of substitution been equal to unity (in which case the capital-labour ratio k_2 would have gone through point E, which would have coincided with point F) relative shares would not have changed, since point B' would have coincided with point A. The same results would be true if the isoquant I_1 had shifted in a parallel way, that is, if technical progress had been HICKS neutral. This can be seen quite easily since, by definition, neutrality implies that along any capital-labour ratio, relative factor-prices have to be constant, otherwise factor shares will not have been constant and HICKS neutrality conditions will not have been satisfied.

This is simply the graphical representation of the phenomenon expressed in equation (10) by the fact that the value of the HICKS bias B equals zero both in the case of no technical progress and in the case of neutral technical progress.

Suppose now that technical progress had been capital-using, i.e., $B > 0$ in equation (10). Assuming that the macroeconomically-determined relative factor price shifted from that determined by the slope of K_1L_1 to that given by the slope of K_2L_2, and that the value of the elasticity of substitution remained the same, i.e. equal to $\frac{FD}{ED}$, then the final effect on relative shares would be made up of two parts. The first, is the shift, given $\sigma = \frac{FD}{ED}$, from the initial relation of $\frac{AL_1}{OA}$ to $\frac{BL_2}{OB} = \frac{B'L_1}{OB'}$, due entirely to the change in relative factor prices. The second part is made up of the biased shift in the isoquant I_1 to I_2, represented by a movement from point G to point M, where isoquant I_2 is tangent to a line (not drawn) parallel to K_2L_2.

The movement from G to M can be further decomposed into two parts. From G to M' the movement is due to the effect of the biased technical change before further factor substitutions take place. In equilibrium, efficiency in production would require, at M', that the price of capital increased relative to wage rates if the capital-labour ratio k_2 were to be maintained. As we have seen, however, at the microeconomic level, it is the capital-labour ratio that is bound to adjust rather than relative factor-prices. Thus, the capital-labour ratio is further increased to k_3, in order to maintain relative factor prices compatible with efficiency in production at point M. The second, movement, therefore, is a factor-substitution effect from M' to M, which at given factor prices can be shown to increase even further the share of capital relative to that of labour.

Summarizing, relative factor shares have shown the following movement: at point D it was equal to $\frac{AL_1}{0A}$; at point G it increased to $\frac{B'L_1}{0B'}$ and finally, at point M it became $\frac{B''L_1}{0B''}$.

Expressions (7) - (10) derived above, can also help clarifying the relationship between HICKS' and other definitions of neutrality.

HARROD's definition of neutrality requires that $\hat{r} = 0$ and $\hat{v} = 0$, where v is the capital-output ratio. Using equations (7) and the fact that $(\hat{v^{-1}}) = T - \Pi_L \hat{k}$ (15), both equated to zero, gives the condition for HARROD capital-using, neutral and labour-using technological change, as follows:

$$B + (1 - \frac{1}{\sigma})\ \hat{k} \gtreqless 0 \left\} \begin{array}{l} \text{capital-using} \\ \text{neutral} \\ \text{labour-using.} \end{array} \right. \qquad (11)$$

As can be seen, if technological change is HICKS neutral (B=0), it will be also HARROD neutral only if $\sigma = 1$. This is the well-known result that, for the COBB-DOUGLAS production function, both HICKS and HARROD neutrality coincide.

A similar index can be found for SOLOW's definition of technological change. Neutrality requires $\hat{w} = 0$ and $\hat{y} = 0$, where y is the output-labour ratio. Using equation (9) and since $\hat{y} = T + \Pi_K \hat{k}$ we acquire the following conditions for SOLOW technological progress;

$$B + (1 - \frac{1}{\sigma})\ \hat{k} \gtreqless 0 \left\} \begin{array}{l} \text{capital-using} \\ \text{neutral} \\ \text{labour-using,} \end{array} \right. \qquad (12)$$

which are the same as HARROD's conditions (11). (16)

(15) See Appendix.

(16) With the production functions defined in such a general form as (3), the concepts of SOLOW and HARROD types of technological change coincide. Actually, they are the same thing replacing L for K and w for r. They are strictly analogous concepts.

b) The Factor-Augmenting Hypothesis:

So far, we have been working with a general form of the production function. For empirical work it is necessary to work out more specific assumption about technological progress. One important form of technological progress is the factor-augmenting hypothesis.(17)

Under this assumption, the general neoclassical production function (3) may be written as

$$Y = F\ (K,L,t) = G\ (a(t)K,\ b(t)L). \tag{13}$$

This specification of the production function is a specialization of the general form and as such, it implies some rather strong restrictions. BURMEISTER et al (1969) present a clear statement of the necessary and sufficient conditions that must be present for factor-augmenting technological change, and discuss the meaning of the implied restrictions in a quite illuminating way.(18)

From the lemma stating such necessary and sufficient conditions they derive three theorems stating that:

a) Technological change is HARROD neutral if, and only if, there exist functions $a(t)$ and $b(t)$ such that F may be represented in the factor-augmenting form with $b(t) \equiv 1$.

b) Technological change is HICKS neutral if, and only if, F may be represented in the factor-augmenting form with $b(t) \equiv a(t)$.

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(17) Among others, BURMEISTER et al (1969, 1970), UZAWA (1961) and BECKMAN et al (1968) have derived conditions for the production function $Y = F\ (K,L,t)$ to admit representation in the factor-augmenting form.

(18) In their 1970 work, they also present a good summary on the importance and implications of factor-augmenting technological progress in modern growth theories. See also SATO (1970).

c) Technological change is SOLOW neutral if, and only if, F may be represented in the factor-augmenting form with $a(t) \equiv 1$.

Assuming G to be linear homogeneous, it can be represented, in intensive form as

$$Y = G\ (a(t)K,\ b(t)L) = b(t)L\ G\ (\frac{a(t)K}{b(t)L},\ 1) = b(t)L\ g(x) \qquad (14)$$

where, $x = \frac{a(t)K}{b(t)L}$ is the factor ratio expressed in factor-augmented form. In other words, the capital-labour ratio is now the ratio of capital to labour measured, both magnitudes, in efficiency units, as opposed to natural units as previously done. Technological progress in factor-augmenting form is represented as if the factors of production had their services increased by the functions $a(t)$ and $b(t)$, although their services, measured in natural units (say man/hours) have remained unchanged.(19) Using the expressions

$$\sigma = \frac{F_K F_L}{F F_{LK}} = \frac{G_1 G_2}{G G_{12}} = -\frac{g'(g-xg')}{xgg''}, \qquad (15)$$

$$\Pi_K = \frac{xg'}{g}, \qquad (16)$$

and the following properties of linear homogeneous functions

$$G \equiv G_1\ a(t)K + G_2\ b(t)L, \qquad (17)$$

$$0 \equiv G_{11}\ a(t)K + G_{21}\ b(t)L, \text{ and} \qquad (18)$$

$$0 \equiv G_{12}\ a(t)K + G_{22}\ b(t)L, \qquad (19)$$

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(19) Increase in efficiency units of the factors of production resulting from factor-augmenting technological change should not be interpreted as meaning improvements in capital or in labour such as education, new design of equipment, embodied results of R & D, etc. Factor-augmenting is a disembodied type of technological progress.

it is possible to differentiate (14) and derive the rate of technical progress and the HICKS bias under the factor-augmenting hypothesis.

$$T = \Pi_K \hat{a} + \Pi_L \hat{b} \tag{4a}$$

$$B = \left(1 - \frac{1}{\sigma}\right) (\hat{a} - \hat{b}) \tag{6a}$$

Expression (6a) defines a HICKS neutral technological change if $(\hat{a} - \hat{b}) = 0$, capital-using technological change if either $(\hat{a} - \hat{b}) > 0$ and $\sigma > 1$, or $\sigma < 1$ and $(\hat{a} - \hat{b}) < 0$, and labour-using technological change if either $\sigma > 1$ and $(\hat{a} - \hat{b}) < 0$ or $\sigma < 1$ and $(\hat{a} - \hat{b}) > 0$.[20]

Using the results obtained so far, and assuming factor-augmenting technological changes, expressions (7) (8) (9) and (10) reduce to

$$\hat{G} = (\Pi_K \hat{K} + \Pi_L \hat{L}) + (\Pi_K \hat{a} + \Pi_L \hat{b}) \tag{7a}$$

$$\hat{r} = \hat{a} - \Pi_L \frac{\hat{x}}{\sigma} = \hat{a} - \frac{\Pi_L}{\sigma} \left[(\hat{a} - \hat{b}) + \hat{k} \right] \tag{8a}$$

$$\hat{w} = \hat{b} + \Pi_K \frac{\hat{x}}{\sigma} = \hat{b} + \frac{\Pi_K}{\sigma} \left[(\hat{a} - \hat{b}) + \hat{k} \right] \tag{9a}$$

$$\hat{S} = \left(1 - \frac{1}{\sigma}\right) \hat{x} = \left(1 - \frac{1}{\sigma}\right) (\hat{a} - \hat{b} + \hat{k}) \tag{10a}$$

(20) Of course, in the COBB-DOUGLAS case, $\sigma = 1$, and technological change is always neutral under HICKS', HARROD's or SOLOW's definition.

Expression (7a) shows that the rate of growth of output can be decomposed into two parts: one due to increases in the amounts of factor services measured in natural units, and the other due to increases in the "efficiency" of such factor services. As before, the distinction is made between a movement along a production function and a shift in it, i.e., technological progress proper.

The growth of factor rewards (equations (8a) and (9a) depends on the factor-augmenting indices (a(t) and b(t)), on the increase in factor services measured in natural units and on the elasticity of factor substitution. Here, different combinations of values for these variables are possible, making it difficult to state general rules relating technological progress and growth of factor rewards. As an example, suppose that technological progress is HICKS neutral, i.e., $\hat{a} - \hat{b} = 0$, and suppose also that in the short-run factor supplies are fixed, i.e., $\hat{k} = 0$. Then $\hat{r} = \hat{w}$ and $\hat{s} = 0$, as we would expect from the definition of HICKS neutrality. Suppose now that $\hat{a} > \hat{b} > 0$. In this case, if $\sigma < 1$ technological change is HICKS labour-using since $B < 0$. As expected, the wage rate would increase and the relative factor share ratio would fall, leading to a more favourable income distribution towards labour. The rate of return on capital, on the other hand, could either increase, remain equal or decrease depending on whether

$$\frac{\hat{a}}{\hat{a} - \hat{b}} \gtreqless \frac{\Pi_L}{\sigma} .$$

In any case, however, labour's income share would increase relative to capital. Allowing for changes in k, all these results could be reversed, showing that only the consideration of specific values for all variables would allow conclusions about the effects of technological change in a dynamic and growing economy.

Finally, it is possible to derive indices for HARROD and SOLOW biases from the expressions derived so far.

HARROD neutral technological change requires a constant capital output ratio ($\hat{v} = 0$) and a constant rate of return on capital ($\hat{r} = 0$). We have seen how these two conditions imply $\hat{S} = 0$, since they are not all independent. Any two of the above conditions imply that the third is equally satisfied. Then $\hat{r} = 0$ implies $\hat{x} = \hat{a}\,\frac{\sigma}{\Pi_L}$; substituting in expression (10a) results in

$$(\sigma - 1)\,\frac{\hat{a}}{\Pi_L} \gtreqless 0 \left.\begin{array}{l}\text{capital-using}\\ \text{neutral}\\ \text{labour-using}\end{array}\right\} . \qquad (11a)$$

Analogously, an index for SOLOW bias is given by

$$(1 - \sigma)\,\frac{\hat{b}}{\Pi_K} \gtreqless 0 \left.\begin{array}{l}\text{capital-using}\\ \text{neutral}\\ \text{labour-using}\end{array}\right\} \qquad (12a)$$

In both cases, technological change is neutral if $\sigma = 1$. If $\sigma \neq 1$, the factor using bias will depend on whether $\sigma \gtrless 1$. For example, if $\sigma < 1$, a HICKS neutral technological change $\hat{a} = \hat{b} > 0$ results in HARROD labour-using and SOLOW capital-using progress; if $\sigma > 1$, then it results in HARROD capital-using and SOLOW labour-using biases.

APPENDIX

In deriving equations (7), (8), (9) and (10) for the two-factor general case, divide equation (4) by F_L, multiply equation (6) by $\frac{1}{y} = \frac{L}{F}$, and adding them, we get

$$\frac{T}{F_L} + \frac{\dot{L}B}{F} = \frac{KF_K F_{Kt} + LF_L F_{Kt}}{FF_L F_K}, \tag{20}$$

from where it follows that

$$\frac{F_{Kt}}{F_K} = T + \Pi_L B \tag{21}$$

$$\frac{F_{Lt}}{F_L} = T - \Pi_K B, \tag{22}$$

where Π_i is the share of total income earned by factor i.

Next, totally differentiating $F_K = F_K(K,L,t)$ gives

$$\dot{F}_K = F_{Kt} + F_{KK}\dot{K} + F_{KL}\dot{L}, \tag{23}$$

and observing that, for linear homogeneous functions, the elasticity of substitution, $\sigma = \frac{F_K F_L}{FF_{KL}}$ and that $- LF_{LL} \equiv KF_{LK}$, and finally that $- KF_{KK} \equiv LF_{KL}$, it follows that

$$\hat{F}_K = \frac{\dot{F}_K}{F_K} = \frac{F_{Kt}}{F_K} - \frac{\Pi_L}{\sigma}\hat{k}. \tag{24}$$

Substituting (21) into (24)

$$\hat{F}_K = \frac{\dot{F}_K}{F_K} = T + \Pi_L \left(B - \frac{\hat{k}}{\sigma}\right), \tag{25}$$

and by a similar procedure,

$$\hat{F}_L = \frac{\dot{F}_L}{F_L} = T - \Pi_K \left(B - \frac{\hat{k}}{\sigma} \right) \tag{26}$$

Assuming competitive markets and profit maximizing behaviour, equations (25) and (26) represent proportional rate of change of the rate of return on capital (r) and of the wage rate (w), respectively. Furthermore, defining v as the capital-output ratio, and M as the marginal rate of technical substitution, it follows that

$$\widehat{(v^{-1})} = T - \Pi_L \hat{k} \text{ , and} \tag{27}$$

$$\hat{M} = \frac{1}{\sigma} \hat{k} - B \tag{28}$$

which enables us to write equations (25) and (26) as

$$\hat{r} = T - \Pi_L \hat{M} = \widehat{(v^{-1})} - \Pi_L \widehat{(S^{-1})} \tag{29}$$

$$\hat{w} = T + \Pi_K \hat{M} = \hat{y} + \Pi_K \widehat{(S^{-1})} \tag{30}$$

where y is the production function (3) expressed in intensive form , $y = f(k,t)$ and $\hat{y} = T + \Pi_K \hat{k}$, and S is the relative income ratio.

Total differentiation of (3) yields

$$\dot{F} = F_t + F_K \dot{K} + F_L \dot{L} \text{ .} \tag{31}$$

Substituting equation (4) and the definitions of, $_K$ and k, one obtains

$$\frac{\dot{F}}{F} = \hat{F} = T + \Pi_K \hat{k} + \hat{L} = T + \Pi_K \hat{K} + \Pi_L \hat{L} \text{ .} \tag{32}$$

Finally, total logarithmic differentiation of Π_K yields

$$\Pi_K = \hat{F}_K + \hat{K} - \hat{F} . \qquad (33)$$

Substituting equations (25) and (32) into (33) yields

$$\hat{\Pi}_K = \Pi_L \left[B + (1 - \frac{1}{\sigma}) \hat{k} \right] . \qquad (34)$$

Analogously,

$$\hat{\Pi}_L = -\Pi_K \left[B + (1 - \frac{1}{\sigma}) \hat{k} \right] , \qquad (35)$$

and using expressions (2), (28), (29) and (30)

$$\hat{S} = \hat{p} + \hat{k} = \hat{r} - \hat{w} + \hat{k} = B + (1 - \frac{1}{\sigma}) \hat{k} . \qquad (36)$$

The following table shows equivalent expressions for two - and "n" - factor cases, in the general and in the factor-augmenting cases.

APPENDIX 1: TABLE OF EXPRESSIONS FOR $\hat{F}$, $\hat{r}$, $\hat{W}$ AND $\hat{S}$, UNDER THE GENERAL AND THE FACTOR-AUGMENTING CASES, FOR 2 AND FOR "N" FACTORS.*

	GENERAL CASE		FACTOR-AUGMENTING CASE	
	$Y = F(K,L,t)$	$Y = F(X_1, X_2, \ldots, X_n, t)$	$Y = F(b(t)L, a(t)K, t)$	$Y = F(a_1(t)X_1, a_2(t)X_2, \ldots, a_n(t)X_n, t)$
$\hat{F} =$	$T + \Pi_K \hat{K} + \Pi_L \hat{L}$	$T + \sum_i \Pi_i \hat{X}_i$	$(\Pi_K \hat{a} + \Pi_L \hat{b}) + (\Pi_K \hat{K} + \Pi_L \hat{L})$	$\sum_i \Pi_i (\hat{X}_i + \hat{a}_i)$
$\hat{r}$	$T + \Pi_L \left(B - \frac{\hat{k}}{\sigma} \right)$	$\hat{F}_m = \begin{cases} \frac{T}{\Pi_m + \Pi_j} + \left(\frac{\Pi_j}{\Pi_m + \Pi_j} \right) B_{mj} - \\ - \sum_{\substack{i=j \\ m \neq i}} \left(\frac{\Pi_i}{\Pi_m + \Pi_j} \right) \frac{F_{it}}{F_i} + \\ + \sum_{i \neq j} \frac{\Pi_i}{\sigma_{mi}} \hat{k}_{ij} \end{cases}$	$\hat{a} - \frac{\Pi_L}{\sigma} \left[(\hat{a} - \hat{b}) + \hat{k} \right]$	$\hat{r}_m = \hat{a}_m + \sum_{i \neq j} \frac{\Pi_i}{\sigma_{mi}} \left[(\hat{a}_i - \hat{a}_j) + \hat{k}_{ij} \right]$
$\hat{W}$	$T + \Pi_K \; B - \frac{\hat{k}}{\sigma}$		$\hat{b} + \frac{\Pi_K}{\sigma} (\hat{a} - \hat{b}) + \hat{k}$	
$\hat{S}$	$B + 1 - \frac{1}{\sigma} \; \hat{k}$	$B_{mj} + \hat{k}_{mj} \left(1 + \frac{\Pi_m}{\sigma_{mm}} + \frac{\Pi_j}{\sigma_{jj}} \right) + \sum_{\substack{i \neq j \\ i \neq m}} \left(\frac{\Pi_i}{\sigma_{mi}} \right) \hat{k}_{ij} - \sum_{\substack{i \neq j \\ i \neq m}} \left(\frac{\Pi_i}{\sigma_{ji}} \right) \hat{k}_{im}$	$\left(1 - \frac{1}{\sigma} \right) \left[(\hat{a} - \hat{b}) + \hat{k} \right]$	$\hat{S}_{mj} = \begin{cases} \left[(\hat{a}_m - \hat{a}_j) + \hat{k}_{mj} \right] \left(1 + \frac{\Pi_m}{\sigma_{mm}} + \frac{\Pi_j}{\sigma_{jj}} \right) + \\ + \sum_{\substack{i \neq j \\ i \neq m}} \frac{\Pi_i}{\sigma_{mi}} \left[(\hat{a}_i - \hat{a}_j) + \hat{k}_{ij} \right] - \\ - \sum_{\substack{i \neq j \\ i \neq m}} \frac{\Pi_i}{\sigma_{ji}} \left[(\hat{a}_i - \hat{a}_m) + \hat{k}_{im} \right] \end{cases}$

* Due to the fact that deriving these expressions involve rather long and tedious demonstrations, they were omitted from the text but are available from the author, on request.

BIBLIOGRAPHY

ALBUQUERQUE, M.C.C. de - A Translog Analysis of Technological Change and Scale Effects in Brazilian Agriculture - A case of Inefficient Modernization - Ph.D. Thesis, Harvard University, University Microfilm International, Ann Arbor, MI, 1985.

ALTMANN, F., KYN, O. and WAGENER, H.J. (eds) - On the Measurement of Factor Productivities: Theoretical and Empirical Results. Papers and Proceedings of the 2nd Reisensburg Symposium, June, 1974. Vandenhoeck & Ruprech, Gottingen, 1976.

ARROW, K. - Economic Welfare and the Allocation of Resources for Invention, in The Rate and Direction of Inventive Activity. Princeton University Press, Princeton, 1962; reprinted in ROSEMBERG (1971).

BECKMANN, M.J. and SATO, R. - Neutral Inventions and Production Functions. Review of Economic Studies, January, 1968.

BLAUG, M. - A Survey of the Theory of Process-Innovations. Económica, February, 1963.

BROWN, M. - Theory and Measurement of Technological Change. Cambridge University Press, Cambridge, 1966.

BURMEISTER, E. and DOBELL, A.R. - Disembodied Technological Change with Several Factors. Journal of Economic Theory, June, 1969.

- Mathematical Theories of Economic Growth. MacMillan Co., New York, 1970.

CORRY, B.A. - The Role of Technological Innovation in Theories of Income Distribution. Papers and Proceedings, American Economic Review, May, 1966.

FEI, J. and RANIS, G. - Innovational Intensity and Factor Bias in the Theory of Growth. International Economic Review 6, 1965.

FERGUSSON, C.E. - The Neo-Classical Theory of Production and Distribution. Cambridge University Press, Cambridge, 1971.

FUSS, M. and MCFADDEN, D. (eds) - Production Economics: A Dual Approach to Theory and Applications. Contributions to Economic Analysis, V.1. North Holland, New York, 1978.

HAMBERG, D. - Production Functions, Innovations and Economi Growth. Journal of Political Economy, June, 1959.

HARROD, R.F. - Towards a Dynamic Economics. MacMillan, London, 1948.

HEERTJE, A. - Economics and Technical Change. Weindenfeld and Nicolson, London, 1973.

HICKS, J.R. - Theory of Wages. MacMillan, London, 1932.

JONES, H.G. - An Introduction to Modern Theories of Economic Growth. McGraw Hill, 1976.

JORGENSON, D.W. - The Embodiment Hypothesis. Journal of Political Ecomomy, V. LXXIV, nº 1, February, 1966.

KENDRICK, J.W. and SATO, R. - Factor Prices, Productivity and Economic Growth. American Economic Review, December, 1963.

KENNEDY, C. - Induced Bias in Innovation and the Theory of Distribution. Economic Journal, LXXIV, 1964.

KENNEDY. C. and THIRLWALL, A.P. - A Surveys in Applied Economics: Technical Progress. Economic Journal, March, 1972.

NADIRI, M.I. - Some Approaches to the Theory and Measurement of Total Factor Productivity: A Survey. Journal of Economic Literature, 1970.

NEHER, P.A. - Economic Growth and Development: A Mathematical Introduction. John Wiley & Sons Inc., New York, 1971.

RESEK, R. - Neutrality of Technical Progress. Review of Economics and Statistics, February, 1963.

ROSEMBERG, N. (ed) - The Economics of Technological Change. Penguin Modern Economics Readings, Baltimore, 1971.

- Perspectives on Technology. Cambridge, University Press, Cambridge, 1976.

RUTTAN, V. - Usher and Schumpeter on Invention, Innovation and Technological Change. Quartely Journal of Economics, Nov, 1959.

SALTER, W.E.G. - Productivity and Technical Change. Cambridge University Press, 1960.

SATO, R. - The Estimation of Biased Technical Progress and the Production Function. International Economic Review, V. 11, nº 2, June, 1970.

SCHMOOKLER, J. - Economic Sources of Inventive Activity. Journal of Economic History, March, 1962.

SHELL, K. Toward a Theory of Inventive Activity and Capital Accumulation Papers and Proceedings. American Economic Review, May, 1966.

SOLOW, R - Technical Change and the Aggregate Production Function. Review of Economics and Statistics, August, 1957.

- Technical Progress, Capital Formation and Economic Growth. Papers and Proceedings, American Economic Review, May, 1962.

UZAWA, H - Neutral Inventions and the Stability of Growth Equilibrium. Review of Economic Studies, 28, 1961.